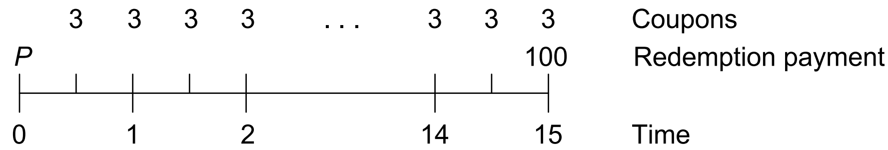
Example 10.3

Company A issues a fixed-interest bond with a term of 15 years. The coupon rate is 6% per annum, and coupons are paid half-yearly in arrears. The bond is redeemable at par.

Calculate the price paid per £100 nominal if the annual effective redemption yield is 8%.

Solution

Based on a nominal value of £100, the annual amount of coupon received is £6, which is paid as coupons of £3 at the end of every six months. The redemption payment is £100. We can display the bond cashflows on a timeline, using P to denote the price per £100 nominal of the bond:



Working in years, and setting the price P equal to the present value of the coupons plus the present value of the redemption payment:

$$P = 6a_{\overline{15}|}^{(2)} + 100v^{15}$$

Evaluating the annuity using an annual effective yield of 8%:

$$a_{\overline{15}|}^{(2)} = \frac{1-v^{15}}{i^{(2)}} = \frac{1-1.08^{-15}}{2\left((1.08)^{0.5}-1\right)} = 8.727375$$

Substituting this into our expression for the price of the bond gives:

$$P = 6 \times 8.727375 + 100(1.08)^{-15} = £83.89$$ ♦♦

As will be the case with other questions on bond pricing, this example involves two percentage rates: 6% and 8%. It can be easy to become confused between the two. 6% is the coupon rate, which determines the amount of the coupon payments. 8% is the annual effective redemption yield, or, equivalently, the rate of return on the investment, which is used to calculate the present value of the cashflows.

We approached Example 10.3 by working in years. An alternative approach would be to use a time unit of 6 months, as this matches the coupon payment frequency. In this case, the term of the bond is 30 half-years, and we would need to use the half-yearly effective yield to evaluate the present value of the cashflows. As the annual effective yield is 8%, the half-yearly effective yield is:

$$(1.08)^{0.5} - 1 = 3.923048\%$$

Working in half-years, the price for £100 nominal of this bond is:

$$P = 3a_{\overline{30}|} + 100v^{30}$$

Using the half-yearly effective yield of 3.923048%:

$$a_{\overline{30}|} = \frac{1-v^{30}}{i} = \frac{1-1.03923048^{-30}}{0.03923048} = 17.454750$$

This gives:

$$P = 3 \times 17.454750 + 100(1.03923048)^{-30} = £83.89$$

as before. So, the price of a fixed-interest bond can be calculated either by working in years, or by working in a time period that matches the frequency of the coupon payment. Both approaches give the same answer. We will work in years for the remainder of the examples in this section.

The basic equation of value used to work out the bond price can also be used to calculate quantities other than the price, as the next two examples illustrate.

Example 10.4

A 7-year bond with an annual coupon rate of 10% pays coupons half-yearly in arrears. The annual effective redemption yield is 4%, and the price paid per £100 nominal is £140.41. Calculate the redemption payment.

Solution

Let the redemption payment be R. Using the basic equation of value for the price of £100 nominal of the bond, we have:

$$140.41 = 10a_{\overline{7}|}^{(2)} + Rv^7$$

Evaluating the annuity using an annual effective yield of 4%:

$$a_{\overline{7}|}^{(2)} = \frac{1-v^7}{i^{(2)}} = \frac{1-1.04^{-7}}{2\left((1.04)^{0.5}-1\right)} = 6.061487$$

Rearranging the equation of value and substituting in the annuity value gives:

$$Rv^7 = 140.41 - 10a_{\overline{7}|}^{(2)} = 140.41 - 10 \times 6.061487 = 79.795133$$

So, the redemption amount is:

$$R = 79.795133(1.04)^7 = £105.00$$

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Example 10.5

An n-year bond pays annual coupons of 5% half-yearly in arrears. The annual effective redemption yield is 8% and the price paid per £100 nominal is £86. The bond is redeemed at 110%. Calculate n.

Solution

The equation of value for the price of £100 nominal of the bond is:

$$86 = 5a_{\overline{n}|}^{(2)} + 110v^n$$

Substituting the formula for $a_{\overline{n}|}^{(2)}$ into the above equation gives:

$$86 = 5\left(\frac{1-v^n}{i^{(2)}}\right) + 110v^n = \frac{5}{i^{(2)}} + \left(110 - \frac{5}{i^{(2)}}\right)v^n$$